ELASTIC DISPLACEMENT OF PRIMARY WAVES FROM EXPLOSIVE SOURCES

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ABSTRACT

Equations derived from Blake's (1952) model of diverging waves from a point source were programmed for computer analysis. Variations of the displacement resulting from changes in the pressure function, propagational velocity, range, and cavity size were investigated. Results of the research indicate that: (1) a step pressure function used with this model gives displacements that closely approximate the displacements measured at 0.4 km from the Gnome nuclear explosion; (2) near the source, long-period displacements are inherent with this model; (3) the periods of the maximum Fourier transform amplitude of the radiation field is proportional to the equivalent cavity radius; (4) the peak displacements scale to the two-thirds power of charge size for values of a between 80 (0.5 kt) and 600 m (275 kt); and (5) between 0.1 and 3.0 cps, the amplitudes of given frequencies scale to the first power of charge size for values of a between 145 (3 kt) and 305 m (28 kt). In general, Fourier amplitudes at frequencies below the natural frequency of the cavity scale to the first power of charge size, and Fourier amplitudes at frequencies above the natural frequency of the cavity scale to a fractional power of charge size. It is suggested that this may be a good model with which to compare near-source observations of seismic phenomena.

Introduction

This investigation compares a theoretical description with results derived from from observed data for compressional seismic waves generated by an explosive source. In this work, theoretical descriptions of ground displacements are considered in the elastic region only, between the ranges of 0.46 and 15.2 kilometers.

Mathematical descriptions of spherically diverging elastic waves instigated by pressure functions have been given by Sharpe (1942), Duvall and Atchison (1950), and Blake (1952). Blake's model was chosen for this investigation. In this model, long-period ground displacements which decay exponentially with the pressure, and oscillatory transients of the radiation field are important for the ranges considered. Very little information regarding long-period displacements is available, but some observations have been made by Weart (1962) and Werth and Herbst (1963).

In the following work, variations of ground displacement have been considered with changes in pressure functions, range, compressional wave propagational velocity, and cavity size.

Analyses

Source

For an infinite homogeneous, isotropic medium, the solution to the wave equation is

$$\varphi = (A/r)f(\tau) \tag{1}$$

where φ is a potential function of displacement, A is a constant, r is range, τ =

$\tau = t - (r - a)/c$, a is the radius of the cavity, t is time, and c is compressional wave propagational velocity.

The boundary condition requires that the radial stress in the cavity wall equals the pressure within the cavity. This is expressed by

$$P(t) = -\rho c^2 \left[\left(\frac{\partial u}{\partial r} \right) + \left(\frac{2\sigma}{1 - \sigma} \right) \left(\frac{u}{r} \right) \right]_{r=a} \tag{2}$$

where: $P(t)$ is the pressure function: ρ is density; u is displacement; and σ is Poisson's ratio, taken to be 0.25 in this work. Blake obtains the following formal solution of the wave equation in terms of displacement potential, φ, for any given pressure function $P(\gamma)$.

$$\varphi = \frac{a^2}{2\pi r \rho c} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{P(\gamma) e^{-jx(\gamma - \tau)(c/a)}}{(x^2 - jx/k - 1/k)} \, dx \, d\gamma \tag{3}$$

where: $x = \omega a/c$; $k = \frac{1}{2}(1 - \sigma)(1 - 2\sigma)^{-1}$; ω is angular frequency; and γ is an integration parameter.

The pressure functions

$$P(\gamma) = P_0(e^{-\alpha_1 \gamma} - e^{-\alpha_2 \gamma}) \qquad \text{for } \gamma \geqq 0$$

and

$$P(\gamma) = 0 \qquad \text{for } \gamma < 0 \tag{4}$$

were used to solve Equation (3). A pressure function of this form allows choice in shaping the pressure by varying the values of α_1 and α_2 between 0 and ∞.

Equation (3) was solved, and it was then differentiated with respect to r to give the following equation for displacement.

$$\begin{aligned} u = {} & \frac{P_0 a}{\rho r[\omega_0^2 + (\alpha_0 - \alpha_1)^2]} \left[e^{-\alpha_1 \tau} \left(\frac{c - r\alpha_1}{rc} \right) + e^{-\alpha_0 \tau} \left\{ -\frac{1}{r} \left(\cos \omega_0 \tau \right. \right. \right. \\ & \left. \left. \left. - \frac{\alpha_0 - \alpha_1}{\omega_0} \sin \omega_0 \tau \right) + \frac{\alpha_1}{c} \cos \omega_0 \tau + \frac{(\alpha_0 - \alpha_1)\alpha_0 + \omega_0^2}{\omega_0 c} \sin \omega_0 \tau \right\} \right] \\ & - \frac{P_0 a}{\rho r[\omega_0^2 + (\alpha_0 - \alpha_2)^2]} \left[e^{-\alpha_2 \tau} \left(\frac{c - r\alpha_2}{rc} \right) + e^{-\alpha_0 \tau} \left\{ -\frac{1}{r} \left(\cos \omega_0 \tau \right. \right. \right. \\ & \left. \left. \left. - \frac{\alpha_0 - \alpha_2}{\omega_0} \sin \omega_0 \tau \right) + \frac{\alpha_2}{c} \cos \omega_0 \tau + \frac{(\alpha_0 - \alpha_2)\alpha_0 + \omega_0^2}{\omega_0 c} \sin \omega_0 \tau \right\} \right] \end{aligned} \tag{5}$$

Equation (5) contains two types of terms: those which have a geometrical divergence of $1/r^2$; and those which have a divergence of $1/r$. Included in the terms whose amplitudes diminish as $1/r^2$ are those which depend upon the exponential decay of the forcing function. These terms represent long-period displacements and, in the case of a pressure stepwave, part of these terms represent permanent displacement. The permanent displacement results from the infinite duration of the pressure on the wall of the cavity.

Equation (5) was programmed for an IBM 1620 Computer to investigate the effect of pressure shape and other factors on the ground displacement of the primary

waves. Figure 1 shows the approximations to the ground displacement measured (Weart, 1962) at 298 meters from the Gnome nuclear explosion. The approximations shown on figure 1 were made using Equation (5), $\alpha_1 = 0$ and $\alpha_2 = \infty$, and assuming that the equivalent radius, a (the radius beyond which the medium behaves elastically), for a one-kiloton yield closely coupled to the medium is 100 meters (Carpenter, Savill, and Wright, 1962). It was assumed that a scales as $W^{1/3}$. Thus, the equivalent radius for Gnome (3 kt) was calculated to be 145

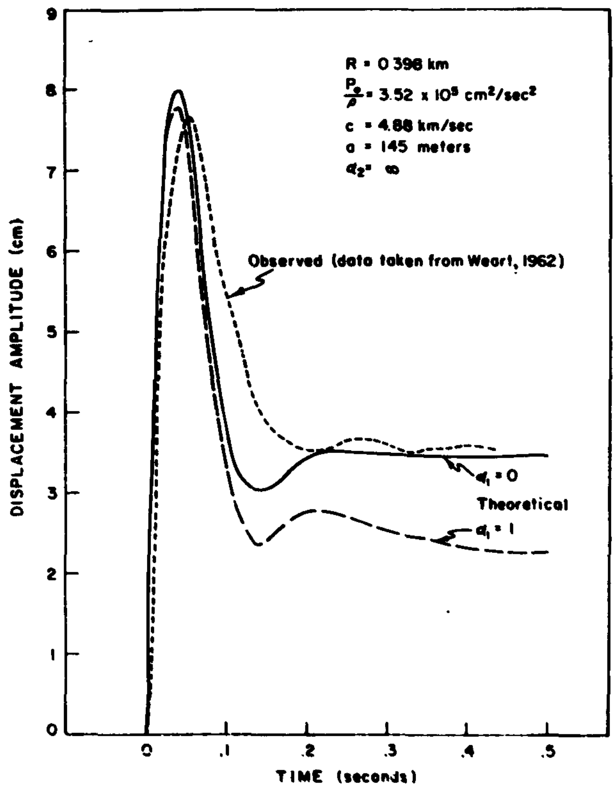

FIG. 1. Comparison of theoretical ground displacement amplitudes with that observed from the Gnome nuclear explosion as given by Weart (1962).

meters. If it was desired, a closer approximation to the observed data could be obtained by changing the values of P_0, α_1, α_2, a, c, and possibly other considerations. From figure 1, the step pressure function [Eq. (5), $\alpha_1 = 0$, $\alpha_2 = \infty$] would be a close approximation to the observed data. This agrees with Latter, LeLevier, Martinelli, and McMillan (1959).

Distance from Source

The displacement function at ranges of 0.46 km, fig. 2A, and 15.2 km, fig. 2B, is plotted for equivalent cavity radii of 145 m (3 kt yield) and 305 m (28 kt yield). For these calculations the pressure was assumed to be a step function. Long-period displacements are evident at the 0.46 km range whereas they are not as evident at

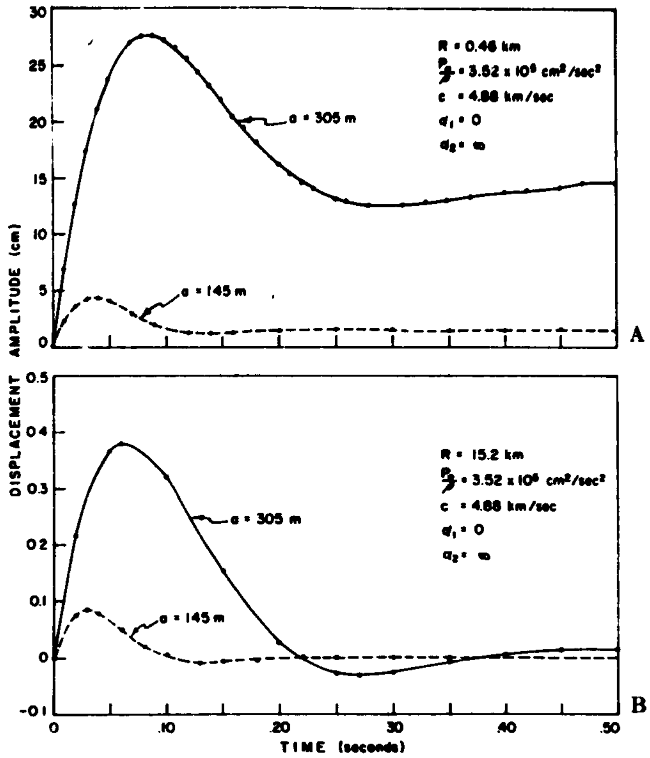

FIG. 2A. Theoretical ground displacements at a range of 0.46 km from the source.
FIG. 2B. Theoretical ground displacements at a range of 15.2 km from the source.

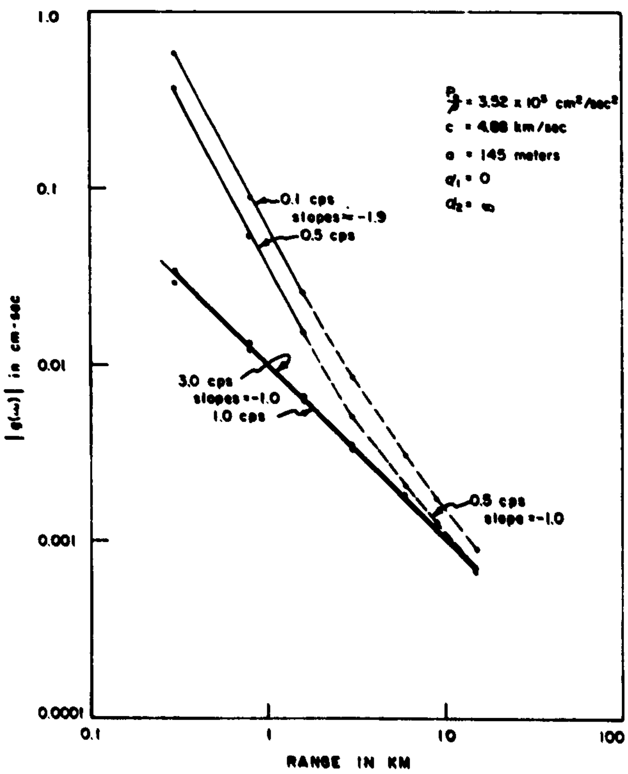

FIG. 3. Fourier Transform Amplitudes for frequencies of 0.1, 0.5, 1, and 3 cps versus range.

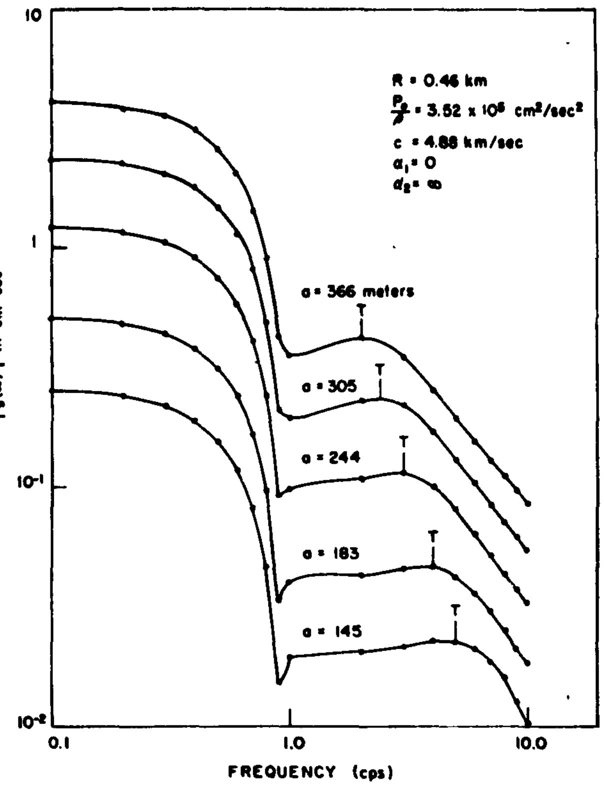

FIG. 4A. Fourier Transform Amplitudes versus frequency for different equivalent cavity radii at a range of 0.46 km.

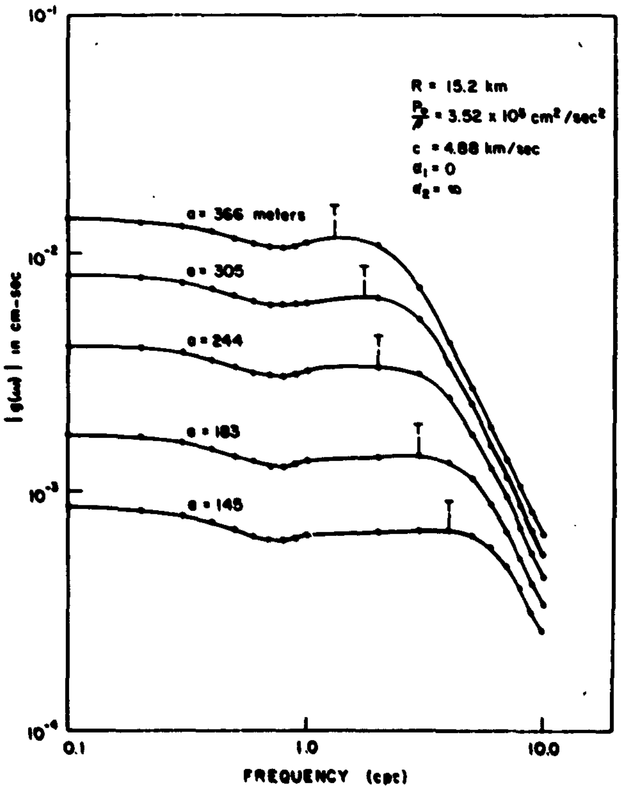

FIG. 4B. Fourier Transform Amplitudes versus frequency for different equivalent cavity radii at a range of 15.2 km.

the 15.2 km range. For computer calculations, all waveforms were truncated after 1 second. It is believed that no serious omissions of data resulted from this operation. The Fourier Transform amplitudes for frequencies of 0.1, 0.5, 1.0, and 3.0 cps are plotted against distance for these ranges. Straight lines are used to represent the decrease of amplitude with distance for 0.5 and 0.1 cps between 0.3 and 1.6 km. The slopes of the line segments are -1.9 cm-sec/km. If the displace-

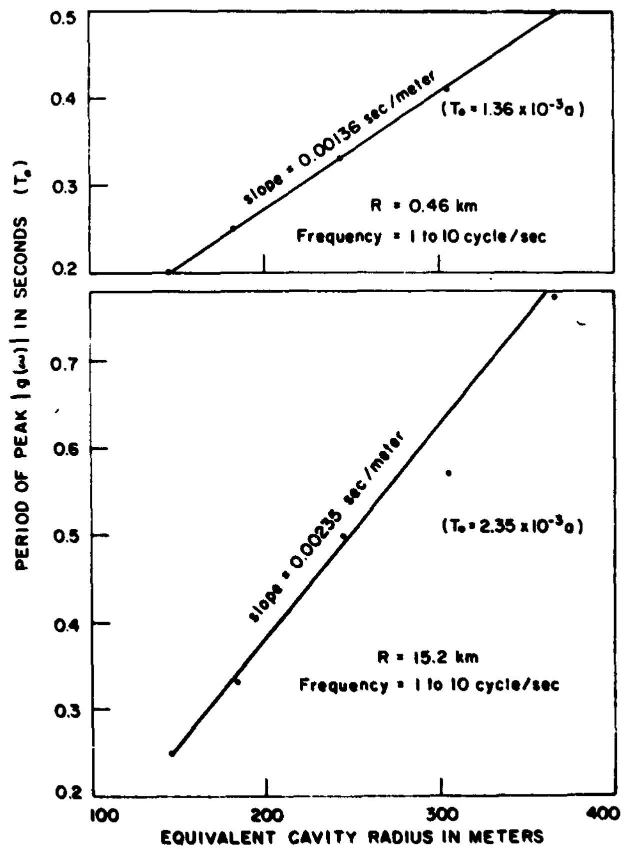

FIG. 5. Period of peak amplitude (see peaks marked T on Figures 4A and 4B) versus equivalent cavity radius.

ment waveforms had not been truncated at 1 second, the amplitudes of the lower frequencies (0.1 and 0.5 cps) would diminish by the inverse square of the range, and at distances greater than 15 km they would diminish as the inverse range [see Equation (5)]. The amplitudes of higher frequencies (1.0 and 3.0 cps) diminish as the inverse range.

Amplitudes, Periods, and Size of Source

The Fourier Transform amplitudes versus frequency are plotted for ranges of 0.46 km (fig. 4A) and 15.2 km (fig. 4B). The long-period displacements are in

evidence for frequencies below about 1 cps; they are considerably smaller at 15.2 km than at 0.46 km from the source. These long-period displacements have been difficult to observe, but they have been observed near nuclear explosions (Weart, 1962, and Werth and Herbst, 1963). However, additional evidence for their existence is desirable.

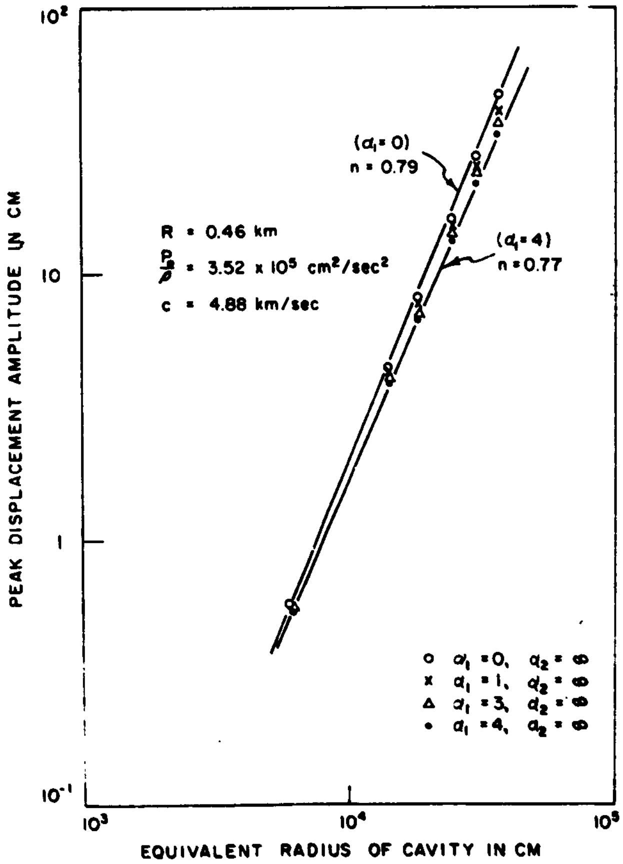

FIG. 6A. Peak displacement amplitude versus equivalent cavity radii for range of 0.46 km, velocity of 4.88 km/sec, and α_1's of 0, 1, 3, and 4, and $\alpha_2 = \infty$.

For frequencies between 1 and 10 cps, the amplitude peaks for the radiation field part of the theoretical source are indicated in figures 4A and 4B by the vertical lines marked "T". It is easy to see the increase in frequency with decreasing a (equivalent cavity radius). If a^3 is proportional to W (charge size), the shift in period of the peak would be expected to be proportional to the equivalent cavity radius, a, as Latter, LeLevier, Martinelli, and McMillan (1959) give the scaling of characteristic frequency, ω_k, as proportional to $W^{-1/3}$. Figure 5 shows a plot of period of peak amplitude versus cavity radius. These variables are proportional at the two distances considered; the slopes of the straight lines vary with distance.

The waveform of ground displacement as given by equation (5) depends upon pressure pulse attenuation, ground velocity, range, and cavity size. For peak ground displacement amplitude, A, the exponent n in the relationship $A \propto W^n$ was investigated for a practical range of the above variables. Figures 6A, 6B, 6C, and 6D show the peak displacement amplitudes plotted against the equivalent

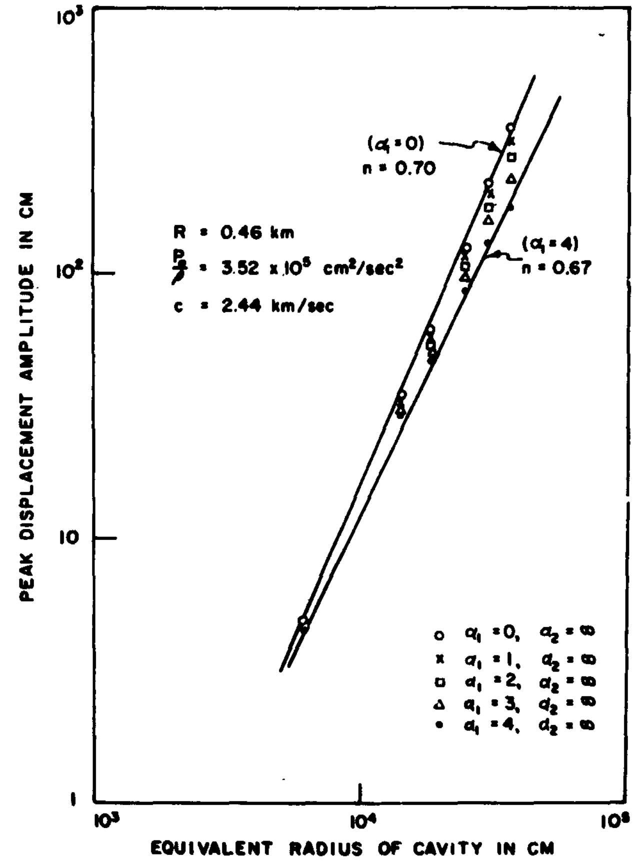

FIG. 6B. Peak displacement amplitude versus equivalent cavity radii for range of 0.46 km, velocity of 2.44 km/sec, and α_1's of 0, 1, 3, and 4, and $\alpha_2 = \infty$.

cavity radii for values of α_1 between 0 and 4, and $\alpha_2 = \infty$. Figures 6A and 6B are computed for velocities of 4.88 and 2.44 km/sec at a range of 0.46 km; figures 6C and 6D at a range of 15.2 km. Computations to compare the values of displacements for different times along the waveform and to read the maximum displacement were performed on a computer. If a^3 is assumed proportional to W, values of n in the relationship $A = kW^n$ are seen to vary between 0.64 and 0.79* for values

* O'Brien (1960) summarized values of the exponent n given by different investigators. The values ranged between 0.73 and 1.39. The largest charge size used to obtain these values was 300 lbs. A value of 0.75 for n was given by Carder and Cloud (1959) for surface waves with periods of 1 to 2 seconds for larger explosions.

of a between 80 and 600 m. This is not a large variation of n. This method of relating the peak displacement amplitude to the size of the source is equivalent to using the peak displacement from the first half cycle of a displacement-type seismogram. These results are comparable to those of Werth and Herbst (1963).

It must be realized that frequency shifts with different charge sizes. Thus, it

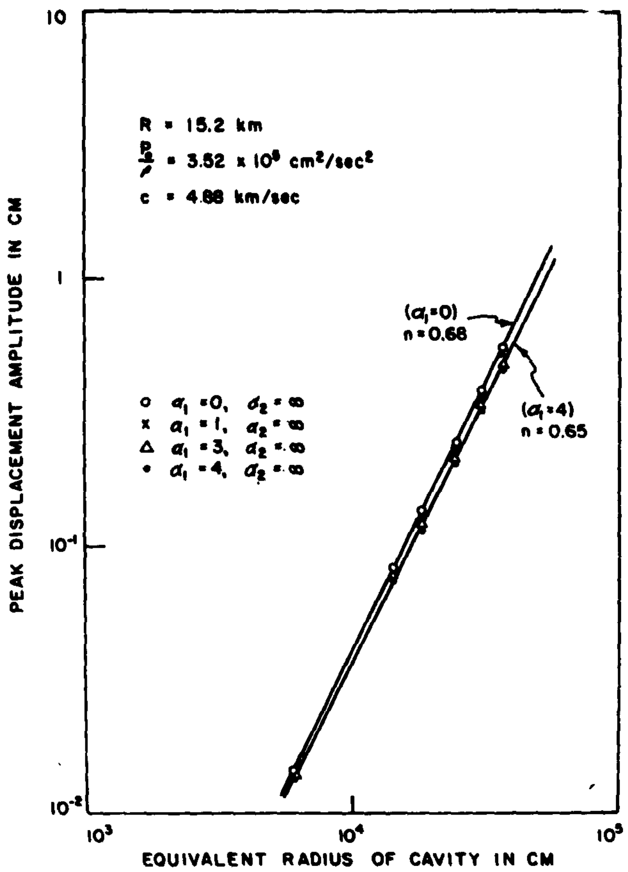

FIG. 6C. Peak displacement amplitude versus equivalent cavity radii for range of 15.2 km, velocity of 4.88 km/sec, and α_1's of 0, 1, 3, and 4, and $\alpha_2 = \infty$.

would be expected that the amplitude of a given frequency would scale differently to charge size than the peak displacement amplitude. The Fourier transform amplitudes versus the equivalent cavity radii are plotted at ranges of 0.46 (fig. 7A) and 15.2 km (fig. 7B). At the 0.46 km range (fig. 7A), the Fourier amplitudes of frequencies between 0.1 and 3 cps scale to the first power of charge size ($n \approx 1$) while those at 5 cps scale to the 0.78 power. The two different amplitude groupings in figure 7A are due to the relative size of the long-period displacements compared to the amplitudes of the radiation field at this range. At the range of 15.2 km (fig. 7B), the Fourier amplitudes between 0.1 and 1 cps scale to the first power of the

charge size while those at 3 and 5 cps are not linear in this region. The natural frequencies of the cavities (f_0) were calculated to be 5 cps and 3 cps for equivalent cavity radii of 145 and 305 meters, respectively, and for a compressional wave velocity of 4.88 km/sec. Thus, for the above range of frequencies, which are below the natural frequencies of the cavities, the Fourier Amplitudes scale to the first

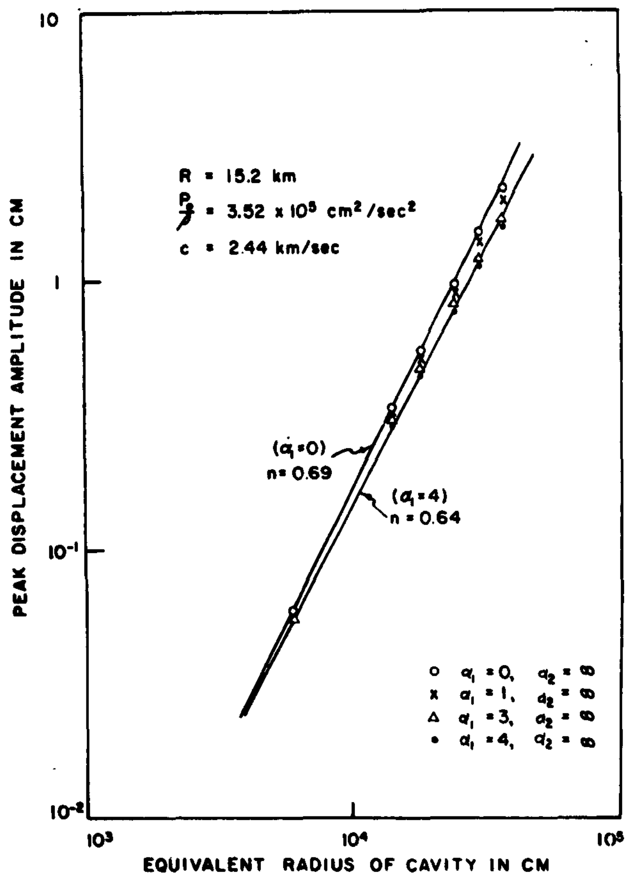

FIG. 6D. Peak displacement amplitude versus equivalent cavity radii for range of 15.2 km, velocity of 2.44 km/sec, and α_1's of 0, 1, 3, and 4, and $\alpha_2 = \infty$.

power ($n \approx 1$) of the charge size. This is in accord with Latter, Martinelli, and Teller (1959). Moreover, an analysis of Equation (4) of their work shows that the Fourier Amplitudes scale to the first power of the charge size for very low frequencies ($\omega < \omega_0$) and to the one-third power of the charge size for high frequencies ($\omega > \omega_0$).*

A value of n, in the relationship $A \propto W^n$, that is different from 0.5 should not be of great concern. The explosion radiates a spectrum of energy, and not energy at one particular frequency in the form of a sine wave. The amplitudes of the different

* In some instances the exponent n may be as large as $\frac{4}{3}$.

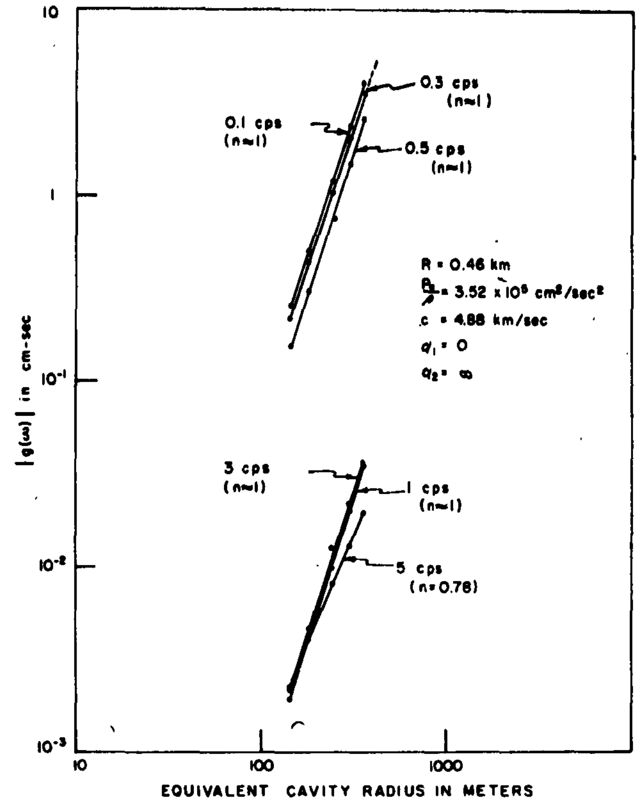

FIG. 7A. Fourier Transform Amplitudes versus equivalent cavity radii for frequencies of 0.1, 0.3, 0.5, 1, 3, and 5 cps at range of 0.46 km.

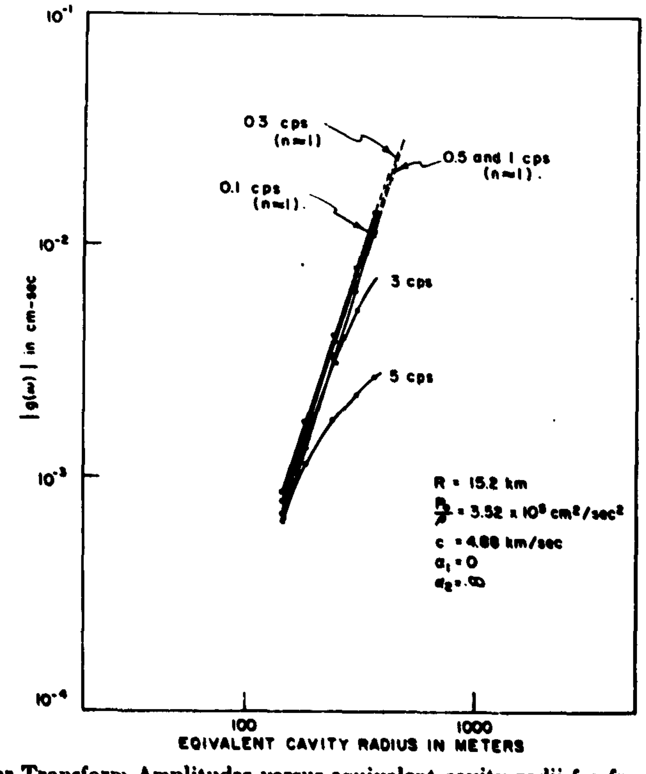

FIG. 7B. Fourier Transform Amplitudes versus equivalent cavity radii for frequencies of 0.1, 0.3, 0.5, 1, 3, and 5 cps at range of 15.2 km.

frequencies scale differently to charge size as shown above. This was pointed out by Peet (1960).

Conclusions

In general, the theoretical source investigated in this research gives results compatible with observations made by other investigators. The results indicate that: (1) a step pressure function used with this model gives displacements that closely approximate the displacements measured at 0.4 km from the Gnome nuclear explosion; (2) near the source, long-period displacements are inherent with this model; (3) the period of the maximum Fourier transform amplitude of the radiation field is proportional to the equivalent cavity radius; (4) the peak displacements scale to the two-thirds power of the charge size for values of a between 80 (0.5 kt) and 600 m (215 kt); and (5) between 0.1 and 3.0 cps, the amplitudes of given frequencies scale to the first power of charge size for values of a between 145 (3 kt) and 305 m (28 kt). In general, Fourier amplitudes at frequencies below the natural frequency of the cavity scale to the first power of charge size, and Fourier amplitudes at frequencies above the natural frequency of the cavity scale to a fractional power of charge size.

The long-period displacements given by this model for near-source ranges offer a problem in that they have been difficult to observe to date. Such displacements could be very important when considering attenuation of energy near the source, generation of surface waves, and other seismic phenomena.

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References

Blake, F. C.
1952. Spherical wave propagation in solid media. *J. Acoustical Soc. Am.*, 24(2): 211–215.
Carder, D. S., and W. K. Cloud
1959. Surface motion from large underground explosions. *J. Geophys. Res.*, 64(10): 1471–1487.
Carpenter, E. W., R. A. Savill, and J. K. Wright
1962. The dependence of seismic signal amplitudes on the size of underground explosions, *Geophys. J. Roy. Astr. Soc.*, 6(4): 426–440.
Duval, W. I. and T. C. Atchison
1950. Vibrations associated with a spherical cavity in an elastic medium. Bur. Mines Rep. of Investigations #692.
Kisslinger, Carl
1963. The generation of the primary seismic signal by a contained explosion, Vesiac. State-of-the-Art Report, Inst. of Science and Technology, University of Michigan, pp. 1–85.
Latter, A. L., E. A. Martinelli, and E. Teller
1959. Seismic Scaling Law for Underground Explosions. *Phys. of Fluids*, 2: 280–282.
Latter, A. L., R. E. LeLevier, E. A. Martinelli, and W. G. McMillan

1959. A method of concealing underground nuclear explosions, R 348 (publication of the Rand Corporation).

O'Brien, P. N. S.
1960. Seismic energy from explosions. *Geophys. J.*, 3(1): 29–44.

Peet, W. E.
1960. A shock wave theory for the generation of the seismic signal around a spherical shot hole. *Geophys. Prospecting*, 8(4): 509–533.

Sharpe, J. A.
1942. The production of elastic waves by explosion pressures, Part I. *Geophys.*, 7(2): 144–154.

Weart, W. D.
1962. Particle motion near a nuclear detonation in Halite. *Bull. Seismol. Soc. Am.*, 52(5): 981–1005.

Werth, G. C., and R. F. Herbst
1963. Comparison of amplitudes of seismic waves from nuclear explosions in four mediums. *J. Geophys. Res.*, 68(5): 1463–1475.

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